Polar coordinates

Introduction

An alternative to using (x, y), or cartesian coordinates, is to use 'polar coordinates'. These are particularly useful for problems involving circular symmetry. This leaflet explains polar coordinates and shows how it is possible to convert between cartesian and polar coordinates.

1. Polar coordinates

When you were first introduced to coordinate systems you will have used *cartesian coordinates*. These are the standard x and y coordinates of a point, P, such as that shown in Figure 1a where the x axis is horizontal, the y axis is vertical and their intersection is the *origin*, O.

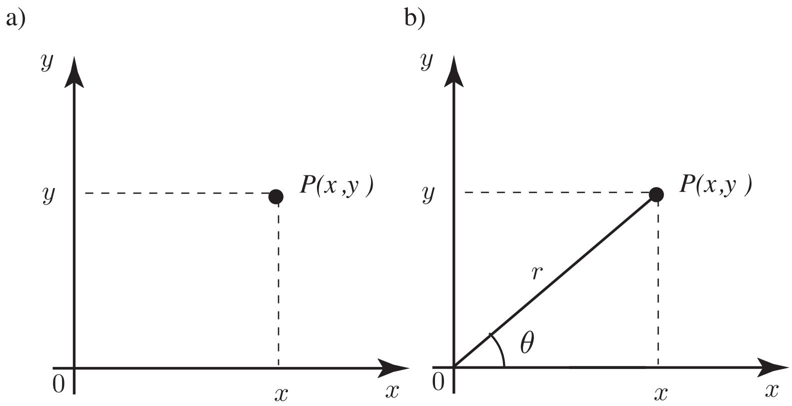

Figure 1. a) Cartesian coordinates, b) Polar coordinates

The position of any point in the plane can be described uniquely by giving its x and y coordinates.

An alternative way of describing the position of a point is to draw a line from the origin to the point as shown in Figure 1b. We can then state the length of this line, r, and the angle, θ between the positive direction of the x axis and the line. These quantities are called the **polar coordinates** of P. It is conventional to denote the polar coordinates of a point either in the form (r, θ), or $r\angle\theta$, although the latter is preferred to avoid confusion with cartesian coordinates. When measuring the angle θ we use the convention that positive angles are measured anticlockwise, and negative angles are measured clockwise. The length of OP is always taken to be positive. Figure 2 shows several points and their polar coordinates.

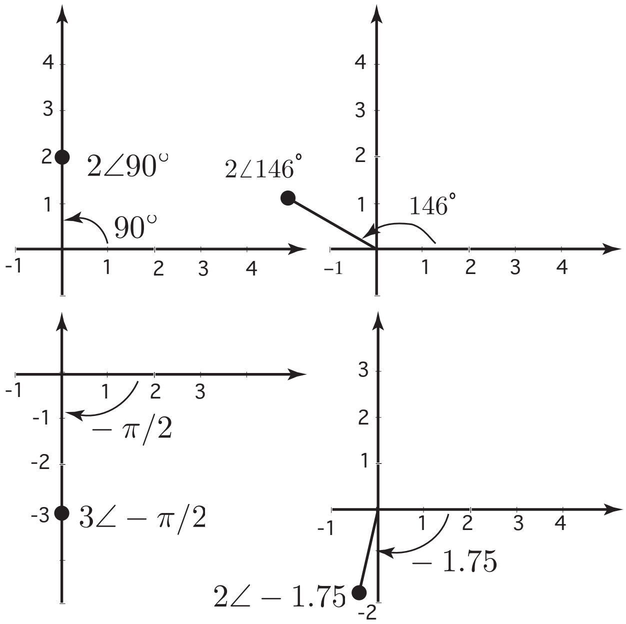

Figure 2. Some points and their polar coordinates

2. Conversion between cartesian and polar coordinates

Look back at Figure 1 b). From trigonometry note that $\cos\theta = \dfrac{x}{r}$ so that $x = r\cos\theta$. Similarly $\sin\theta = \dfrac{y}{r}$ so that $y = r\sin\theta$. Hence if we know the polar coordinates of a point $r\angle\theta$, we can find its cartesian coordinates.

Alternatively, using Pythagoras' theorem note that $r = \sqrt{x^2 + y^2}$. Further, $\tan\theta = \dfrac{y}{x}$ so that $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$. However, when calculating θ you should take special care to ensure that θ is located in the correct quadrant. The result produced by your calculator can be misleading. A diagram should always be sketched and will help you decide the correct quadrant.

$$x = r\cos\theta, \qquad y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}, \qquad \tan\theta = \frac{y}{x}$$

Exercises

In each case sketch a diagram showing the point in question. Angles in degrees are denoted by the degrees symbol °. Otherwise assume that the angle is measured in radians.

1. Calculate the cartesian coordinates of the following points.

a) $3\angle 2$, b) $4\angle 0.7$, c) $1\angle 180^\circ$

2. Calculate the polar coordinates of the following points.

a) $(3, 4)$, b) $(-2, 1)$, c) $(-2, -3)$.

Answers

1. a) $(-1.25, 2.73)$, b) $(3.06, 2.58)$, c) $(-1, 0)$. 2. a) $5\angle 0.927$, b) $\sqrt{5}\angle 2.678$, c) $\sqrt{13}\angle -2.159$